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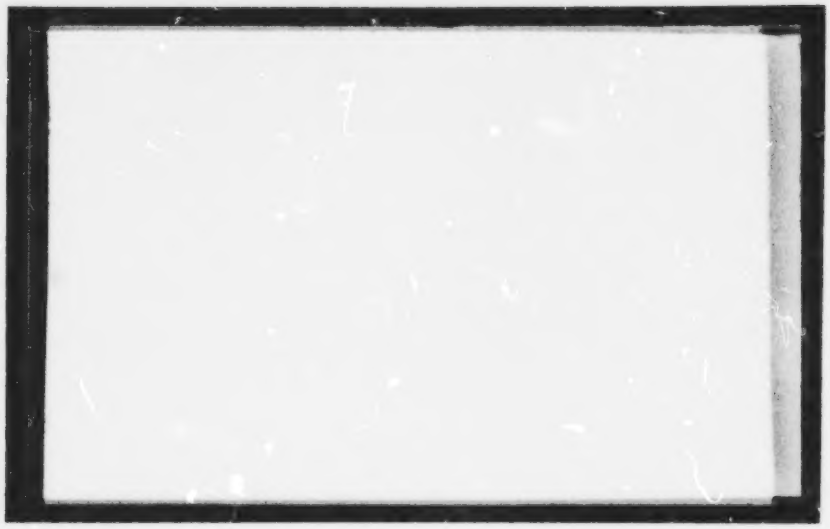

CENTRE DE RECHERCHE SUR LES TRANSPORTS

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A SIMPLEX-TYPE APPROACH TO SOLVE THE ENGINE SCHEDULING PROBLEM

by

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ABSTRACT

In this paper we explore several variants of Benders' decomposition to solve a scheduling problem that is faced by any railway company using several engine types to provide power for its trains. The numerical experimentation indicates the superiority of a new "simplex-type" approach over several other variants of Benders' decomposition.

1. INTRODUCTION

In this paper we explore several variants of Benders' decomposition to solve a scheduling problem that is faced by any railway company using several engine types to provide power for its trains. The problem is to select the mix of engine types for each train that gives the lowest capital investment and operating costs. In [4], a straightforward Benders' decomposition has been shown inefficient to solve it. Here, a new simplex-type approach is used and compared with several other variants of Benders' decomposition. It is itself a variant of Benders' decomposition. The numerical experimentation indicates its superiority.

In Section 2, the problem and the model are summarized. Several variants of Benders' decomposition, including the one introduced by Geoffrion and Graves in [7], are used without any success to solve the problem. Then, in Section 4, the new simplex-type approach is presented.

2. THE PROBLEM AND THE MODEL

The problem and the mathematical model for it have been analyzed in details in [4]. This section summarizes this presentation.

Assume that the trains schedule is known. A convenient way to represent graphically the mouvement of the trains is to use a time space diagram. This diagram is a network $G=(N,A)$ where each node is characterized by two coordinate: the horizontal coordinate is to specify the time and the vertical coordinate is to specify the geographical location of the city. Train arcs are diagonal arcs in the network, and horizontal

arcs are added to connect nodes of diagonal arcs terminating or originating at the same city (See Figure 1, taken from [4]).

Assume also that the railway company employs several engine types to provide power for its trains. Let $I \geq 1$ be the number of engine types available. The flows associated with each diagonal arc are the engines assigned to the corresponding train, and associated with horizontal arc the engines that are idle and available to be reassigned to other trains originating later at the same city.

On the one hand, each engine type i can produce a maximum tractive effort $s_i(x,y)$ and has a horsepower rating $w_i'(x,y)$. For practical reasons, $s_i(x,y)$ may take alternate values for different trains (x,y) (see [4] for details) although $w_i'(x,y)$ is the same for all trains (x,y). On the other hand, each train (x,y) requires a minimun tractive effort T(x,y) and a minimum horsepower rating WP(x,y) to be moved. This generates two constraints for each train (x,y).

$$\sum_{i=1}^{I} s_i(x,y)\ell_i(x,y) \geq T(x,y)$$

$$\sum_{i=1}^{I} w_i'(x,y)\ell_i(x,y) \geq WP(x,y)$$

where $\ell_i(x,y)$ is the number of engines of type i providing power for trains (x,y). For convenience, let $w_i'(x,y) = w_i(x,y)\frac{WP(x,y)}{T(x,y)}$, and then the second constraint becomes

$$\sum_{i=1}^{I} w_i(x,y)\ell_i(x,y) \geq T(x,y) .$$

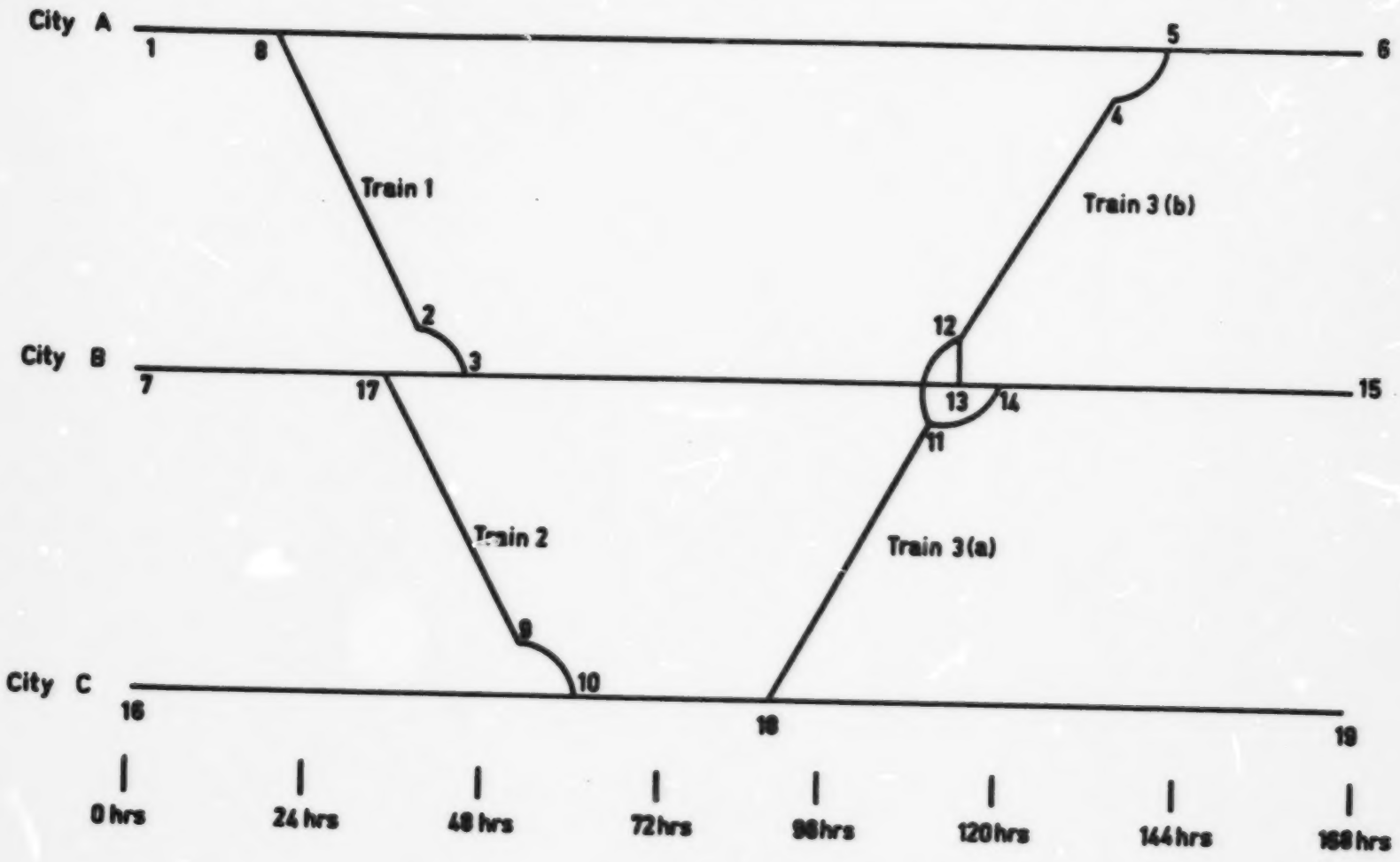
City A
City B
City C
1
8
5
6
4
Train 1
Train 3 (b)
2
12
3
7
17
13
14
15
11
Train 2
Train 3(a)
9
10
16
18
19
0 hrs
24 hrs
48 hrs
72 hrs
96 hrs
120 hrs
144 hrs
168 hrs

Figure 1

In addition, the engines must circulate in the railway network to prevent imbalances that may cause short supply at some cities. Hence, for each city, the network includes an arc that originates at the last time period and terminates at the first time period. This arc ensures that the engines available at one city at the end of a planning period are available at the same city at the beginning of the next planning period.

Thus, if $f_i(x,y)$ denotes the number of engines of type i flowing on arc (x,y), and if $A_t \subset A$ denotes the set of train arcs, then the constraints of the problem are

$$\left.\begin{aligned} &\sum_{i=1}^{I} s_i(x,y)\ell_i(x,y) \geq T(x,y) \quad (2.1) \\ &\sum_{i=1}^{I} w_i(x,y)\ell_i(x,y) \geq T(x,y) \quad (2.2) \end{aligned}\right\} \quad , (x,y)\in A_t \ ,$$

$$f_i(x,N) - f_i(N,x) = 0 \quad , \quad x\in N \ , \ 1 \leq i \leq I \ , \tag{2.3}$$

$$f_i(x,y) \geq \ell_i(x,y) \quad , \ (x,y)\in A_t, \ 1 \leq i \leq I \ , \tag{2.4}$$

$$f_i(x,y), \ \ell_i(x,y) \geq 0, \text{ integer} \ , \ (x,y)\in A \ , \ 1 \leq i \leq I. \tag{2.5}$$

The constraints (2.3) are the flow conservation constraints, and constraints (2.4) insure a sufficiently large flow of engines in each train arc to provide power for each train.

The cost structure is specified as follows. A linear investment cost $a_i(x,y)$ is charged for each engine of type i flowing in arc (x,y), and for each train arc, and additional operating cost $a_i'(x,y)$, is charged for each engine of type i providing power for train (x,y). The objective

function is then

$$\text{Min } \{ \sum_{i=1}^{I} \sum_{(x,y)\in A} a_i(x,y) f_i(x,y) + \sum_{i=1}^{I} \sum_{(x,y)\in A_t} a_i'(x,y) \ell_i(x,y) \}$$

Finally, for each train arc (x,y) let Q_{xy} denote the set of feasible undominated mix of engines satisfying the constraints

$$\sum_{i=1}^{I} s_i(x,y) \ell_i(x,y) \geq T(x,y)$$

$$\sum_{i=1}^{I} w_i(x,y) \ell_i(x,y) \geq T(x,y)$$

$$\ell_i(x,y) \geq 0, \quad \text{integer}, \quad 1 \leq i \leq I.$$

This set includes I-tuple $[q_1^j(x,y), q_2^j(x,y), \ldots, q_I^j(x,y)]$ where $q_i^j(x,y)$ is the number of engines of type i in the j^{th} mix. A mix j is undominated if there is no other feasible mix j' such that $q_i^{j'}(x,y) \leq q_i^j(x,y)$ for all $1 \leq i \leq I$ and $q_i^{j'}(x,y) < q_i^j(x,y)$ for some i, $1 \leq i \leq I$. A dominated mix has to be eliminated because it increases the total cost. A technique to determine this set is analyzed in [4]. Let $|Q_{xy}|$ denotes the number of elements in Q_{xy}.

With the j^{th} mix in Q_{xy} associate the 0-1 variable $z_j(x,y)$. Then constraints (2.1) and (2.3) are replaced by

$$\sum_{j=1}^{|Q_{xy}|} z_j(x,y) = 1 \quad , \quad (x,y)\in A_t$$

$$z_j(x,y) = 0 \text{ or } 1 \quad , \quad 1 \leq j \leq |Q_{xy}| \ , \ (x,y)\in A_t,$$

and $\ell_i(x,y) = \sum_{j=1}^{|Q_{xy}|} z_j(x,y) q_i^j(x,y)$. Thus substituting the value of $\ell_i(x,y)$ in (2.4) and in the objective, the problem becomes

$$\text{Min } \{ \sum_{i=1}^{I} \sum_{(x,y)\in A} a_i(x,y) f_i(x,y) + \sum_{(x,y)\in A_t} \sum_{j=1}^{|Q_{xy}|} c_j(x,y) z_j(x,y) \}$$

subject to

$$f_i(x,N) - f_i(N,x) = 0 , \quad x\in N, \ 1 \le i \le I,$$

$$f_i(x,y) - \sum_{j=1}^{|Q_{xy}|} z_j(x,y) q_i^j(x,y) \ge 0 , \quad (x,y)\in A_t, \ 1 \le i \le I, \tag{2.6}$$

$$\sum_{j=1}^{|Q_{xy}|} z_j(x,y) = 1 , \quad (x,y)\in A_t,$$

$$f_i(x,y) \ge 0, \text{ integer} , \quad (x,y)\in A, \ 1 \le i \le I,$$

$$z_j(x,y) = 0 \text{ or } 1 , \quad 1 \le j \le |Q_{xy}|, (x,y)\in A_t,$$

where $c_j(x,y) = \sum_{i=1}^{I} a_i'(x,y) q_i^j(x,y)$. Notice that this problem is an approximation of the original problem since we should only require that $z_j(x,y) \ge 0$, $1 \le j \le |Q_{xy}|$, $(x,y)\in A_t$ and that $\sum_{j=1}^{|Q_{xy}|} z_j(x,y) q_i^j(x,y)$ be integer, $(x,y)\in A_t$, $1 \le i \le I$. But the constraints $z_j(x,y) = 0$ or 1, $1 \le j \le |Q_{xy}|$, $(x,y)\in A_t$ are easier to handle.

3. DECEIVING EARLIER EXPERIENCE WITH BENDERS' DECOMPOSITION

The structure of this problem suggests the application of Benders' decomposition since once the variables $z_j(x,y)$ are determined, the sub-problem in $f_i(x,y)$ can be decomposed into I circulation problems independent

of each other. Its adaptation to our problem is as follows:

Step 0: Select a tolerance parameter $\varepsilon \geq 0$. Initialize $UB = \infty$, $LB = -\infty$, $K = 1$. Determine a feasible mix in Q_{xy} for each $(x,y) \in A_t$, i.e. for each $(x,y) \in A_t$ determine j_{xy}, $1 \leq j_{xy} \leq |Q_{xy}|$, such that $z_{j_{xy}}(x,y) = 1$ and $z_j(x,y) = 0$ for all $j \neq j_{xy}$. Move to Step 2.

Step 1: Solve the problem

$$\text{Min } y_o$$

Subject to

$$y_o - \sum_{(x,y)\in A_t} \sum_{j=1}^{|Q_{xy}|} [c_j(x,y) + \sum_{i=1}^{I} \mu_i^k(x,y) q_i^j(x,y)] z_j(x,y) \geq 0, \quad 1 \leq k \leq K$$

$$\sum_{j=1}^{|Q_{xy}|} z_j(x,y) = 1, \quad (x,y) \in A_t$$

$$z_j(x,y) = 0 \text{ or } 1, \quad 1 \leq j \leq |Q_{xy}|, \quad (x,y) \in A_t.$$

Let LB equal to the optimal value of this problem, and j_{xy} be the index of the variable $z_{j_{xy}}(x,y)=1$, $(x,y) \in A_t$. Put $K=K+1$, and move to step 2.

Step 2 Solve the problems

$$\text{Min } \sum_{(x,y)\in A} a_i(x,y) f_i(x,y)$$

Subject to

$$f_i(x,N) - f_i(N,x) = 0, \quad x \in N,$$

$$f_i(x,y) \geq q_i^{J_{xy}}(x,y), \quad (x,y) \in A_t \tag{3.1}$$

$$f_i(x,y) \geq 0, \quad (x,y) \in A$$

for each i, $1 \le i \le I$. Let $\{\bar{f}_i(x,y):(x,y)\in A\}$ be an optimal solution, and $\mu_i^k(x,y)$ optimal multipliers associated with constraints (3.1).

Take $UB = \text{Min}\{UB, \sum_{(x,y)\in A} a_i(x,y)\bar{f}_i(x,y) + \sum_{(x,y)\in A_t} c_{j_{xy}}(x,y)\}$.

If $UB-LB>\varepsilon$, move to Step 1. Otherwise, terminate with the ε-optimal solution on hand.

In our numerical experimentation with this algorithm, the 0-1 programming problem in Step 1 is solved with a very efficient heuristic yielding a suboptimal solution [3], and the circulation problems in Step 2 are solved with the Out-of-Kilter algorithm [6] as coded in [2].

The approach was tested on the Canadian National Railways network for the central region going from Montreal to Winnipeg. It includes 1542 nodes, 1931 arcs of which 379 are train arcs. Two engine types are used.

The results obtained with this algorithm were very deceiving. Table 1 indicates that it was not possible to improve the initial solution even after 20 iterations. Furthermore the difference $\frac{UB-LB}{UB} = 0{,}097$ is still very large. Notice that since the algorithm to solve the 0-1 programming problem is suboptimal, LB is the largest value over all iterations of Step 1 and the real difference is larger than 0,097.

Then the algorithm was modified to incorporate the variant mentioned by Geoffrion and Graves in [7]. The basic idea is to stop the resolution of the problem in Step 1 before optimality is reached. Instead, the resolution

is stopped whenever the value of y_0 becomes smaller than UB-ρ. This variant was tested with two sets of values for ρ. The first set (called "smaller ρ") has ρ=(0,10)UB for the first two iterations, ρ=(0,07)UB up to the 8^{th} iteration, ρ=(0,05)UB up to the 12^{tj} iteration, and ρ=(0,03)UB afterward. The set "larger ρ" has ρ=(0,20)UB for the first two iterations, ρ=(0,13)UB up to the 8^{th} iteration, ρ=(0,09)UB up to the 12^{th} iteration, and ρ=(0,05)UB afterward. The results in Table 2 indicate that it has the same pitfalls as the original algorithm.

A third variant was tested. It originates from the conjecture that the inefficiency of the original-algorithm was due to the small number of constraints (referred to as Benders' cuts) in problem of Step 1. Thus at each iteration this problem has a maximum number of constraints. In our example, the computer memory available reduced this maximum number to 32. Furthermore, since circulation problems are highly degenerated, it is fairly easy to determine several Benders' cuts at each iteration. The strategy was to generate 16 new cuts at each iteration and to keep the 16 most binding constraints among the 32 at the preceding iteration. The results in Table 3 are even worst than with the preceding variants.

These failures with Benders' decomposition motivated the effort of the authors to define a new approach.

Iteration number	Value of the solution in Step 1	Value of the solution in Step 2
		278972
1	108495	452002
2	240892	329969
3	249111	304029
4	251589	359633
5	240992	332653
6	251793	312980
7	251033	306792
8	246008	305980
9	241385	306433
10	242917	303676
11	248324	312016
12	245372	306371
13	246984	318712
14	250245	317845
15	250668	328833
16	248150	310786
17	245318	311465
18	249200	304135
19	249480	296411
20	249626	305355
	$\frac{UB-LB}{UB} = 0,0974$	

TABLE 1

	larger ρ		smaller ρ	
Iteration number	Value in Step 1	Value of the solution in Step 2	Value in Step 1	Value of the solution in Step 2
		278972		278972
1	247313	274714	222345	303601
2	246884	299311	222879	314141
3	255451	299228	242431	318958
4	255246	310472	242514	305988
5	255279	299467	242434	301906
6	255250	293812	242704	305110
7	255248	287403	241870	322396
8	255461	289723	242050	319908
9	260812	290390	250680	323974
10	260681	292611	251064	312889
11	260946	283818	250684	304589
12	260960	291851	251054	295842
13	265442	291083	265007	304438
14	266262	295034	264943	306920
15	265695	291296	264557	303034
16	266022	285057	265001	292600
17	266011	285608	264381	299374
18	266437	286859	264366	295352
19	266465	292247	264983	304501
20	181012	347698	225442	310947
	$\frac{UB-LB}{UB} = 0,3411$		$\frac{UB-LB}{UB} = 0,1919$	

TABLE 2

Iteration number	Value of the solution in Step 1	Value of the solution in Step 2
		278972
1	155139	412451
2	235199	342406
3	211369	395919
4	205524	401636
5	207259	377692
6	199127	403992
7	201093	374083
8	226110	383913
9	210391	377518
10	217722	371885
	$\frac{UB-LB}{UB} = 0,2196$	

TABLE 3

4. NEW APPROACH EQUIVALENT TO A VARIANT OF BENDERS' DECOMPOSITION

Let us first introduce the approach which is very intuitive, and then show it to be a variant of Bender's decomposition.

Referring to problem (2.6), once a feasible set of values for the variables $z_j(x,y)$ is available, it reduces to I circulation problems, one for each commodity. These problems can be solved by the Out-of-Kilter algorithm, and optimal multipliers $\mu_i(x,y)$ associated with constraints (3.1) are also easily obtained. Using these multipliers and defining $\tau(x,y)$ the multiplier associated with the constraint $\sum_{j=1}^{|Q_{xy}|} z_j(x,y) = 1$, each variable $z_j(x,y)$ can be priced-out as in the simplex algorithm. The relative cost $\bar{c}_j(x,y)$ of $z_j(x,y)$ is then

$$\bar{c}_j(x,y) = c_j(x,y) + \sum_{i=1}^{I} \mu_i(x,y) q_i^j(x,y) - \tau(x,y). \tag{4.1}$$

Now, recall that a set of feasible values for the variables $z_j(x,y)$ corresponds to selecting exactly one mix j_{xy} for each train arc (x,y). Let

$$\tau(x,y) = c_{j_{xy}}(x,y) + \sum_{i=1}^{I} \mu_i(x,y) q_i^{j_{xy}}(x,y) \quad , \quad (x,y) \in A_t \ . \tag{4.2}$$

Then $\bar{c}_{j_{xy}}(x,y) = 0$, and if $\bar{c}_j(x,y) < 0$ for some $1 \le j \le |Q_{xy}|$, then the objective function decreases when the value of $z_j(x,y)$ increases. This is a "simplex-type" approach. Unfortunately, since $z_j(x,y) = 0$ or 1, then if the value of $z_j(x,y)$ increases, $z_j(x,y)$ takes the value 1. But the objective function might increase when $z_j(x,y) = 1$. So provisions have to be made to take this in consideration, and the following heuristic algorithm can be used to solve problem (2.6).

Step 0: Initialize UB=∞. Determine a feasible mix in Q_{xy} for each $(x,y)\epsilon A_t$, i.e. for each $(x,y)\epsilon A_t$ determine j_{xy}, $1 \le j_{xy} \le Q_{xy}$, such that $z_{j_{xy}}(x,y) = 1$ and $z_j(x,y) = 0$ for all $j \neq j_{xy}$. Move to Step 1.

Step 1: Solve the problems

$$\text{Min} \sum_{(x,y)\epsilon A} a_i(x,y) f_i(x,y)$$

Subject to

$$\begin{aligned} f_i(x,N) - f_i(N,x) &\quad 0 \quad , \ x\epsilon N \\ f_i(x,y) &\ge q_i^{j_{xy}} \quad , \ (x,y)\epsilon A_t \\ f_i(x,y) &\ge 0 \quad , \ (x,y)\epsilon A \end{aligned} \tag{4.3}$$

for each i, $1 \le i \le I$. Let $\{\bar{f}_i(x,y):(x,y)\epsilon A\}$ be an optimal solution, and $\mu_i(x,y)$ optimal multipliers associated with (4.3). If $UB \le \sum_{(x,y)\epsilon A} a_i(x,y)\bar{f}_i(x,y) + \sum_{(x,y)\epsilon A_t} c_{j_{xy}}(x,y)$, then move to Step 3. Otherwise, $UB = \sum_{(x,y)\epsilon A} a_i(x,y)\bar{f}_i(x,y) + \sum_{(x,y)\epsilon A_t} c_{j_{xy}}(x,y)$, and move to Step 2.

Step 2: Determine for all $1 \le j \le Q_{xy}$ and for all $(x,y)\epsilon A_t$

$$\bar{c}_j(x,y) = c_j(x,y) - c_{j_{xy}}(x,y) + \sum_{i=1}^{I} \mu_i(x,y)(q_i^j(x,y) - q_i^{j_{xy}}(x,y)).$$

Let $\Gamma_{xy} = \emptyset$ for all $(x,y)\epsilon A_t$. Move to Step 3.

Step 3: Determine

$$\Delta = \min_{(x,y)\in A_t} \{ \min_{\substack{1 \le j \le |Q_{xy}| \\ j \notin \Gamma_{xy}}} \{\bar{c}_j(x,y)\}\} .$$

with the convention that the Min over an empty set equal ∞. If $\Delta \ge 0$, then terminate. If $\Gamma_{xy} = \emptyset$ for all $(x,y)\in A_t$, then the solution is optimal, otherwise it is localy optimal. If $\Delta < 0$, let $\bar{c}_{\bar{j}}(\overline{x,y}) = \Delta$. Then, for train arc $(\overline{x,y})$, the $\bar{j}$ mix replaces the $j_{\overline{xy}}$mix; i.e. $j_{\overline{xy}} \leftarrow \bar{j}$. Let $\Gamma_{\overline{xy}} = \Gamma_{\overline{xy}} \cup \{\bar{j}\}$, and move to Step 1.

This heuristic approach has the virtue that the value of the objective function decreases each time the relative costs are modified. Notice also that moving from Step 3 to Step 1 implies a modification for the bound of only one arc in each circulation problem. Each of these problems is easily reoptimized since only few iterations are necessary, only one arc beign (possibly) Out-of-Kilter.

Notice the similarity between the relative cost $\bar{c}_j(x,y)$ defined in relation (4.1) and the Benders' cut of the problem in Step 1 (in the algorithm of Section 3). Thus the new approach can be viewed as a variant of the Benders' decomposition where only the last cut generated is used to modify the variables $z_j(x,y)$. Furthermore only two variables are modified (one changes from 0 to 1 and the other from 1 to 0) and the new algorithm evaluates this modification before changing the solution.

Another variant of this approach is to modify more than two variables at the same time (20 variables for instance), but the numerical experimentation indicates the superiority of the original variant.

Finally, the approch was successful when starting with a good initial solution. Starting at a random solution generates slow convergence.

The same problem was solved with this heuristic. Starting with a solution with a value equal to 278972 (as before), the algorithm generated a local optimal solution with a value equal to 274046 in 405 sec. of C.P.U. (on the Cyber 173 at the Université de Montréal). It took about 1000 sec. more of C.P.U. to complete the algorithm in order to verify that the solution is a local minimum. Notice that the different variants of Benders' decomposition require about the same amount of C.P.U. time to complete 20 iterations for this problem.

5. CONCLUSION

This new simplex-type approach to solve the problem is more efficient than several other variants of Benders decomposition including the one used by Geoffrion and Graves [7] to solve a multicommodity distribution system. It seems to indicate that the efficiency of Benders' decomposition is problem dependent in the sense that the technique has to be adapted to each problem.

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